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Leading electroweak logarithms at one loop

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We summarize results for the complete one-loop electroweak logarithmic corrections for general processes at high energies and fixed angles. Our results are applicable to arbitrary matrix elements that are not mass-suppressed. We give explicit results for W-boson-pair production in e^+e^- annihilation.

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1 Introduction

Future colliders, such as the LHC [1] or an $\mathrm{e^+e^-}$ linear collider (LC) [2], will explore the energy range $\sqrt{s} \gg M_\mathrm{Z}$. It is known since many years (see, for instance, Refs. [3,4]) that above the electroweak scale the structure of the leading electroweak corrections changes and double logarithms of Sudakov type [5] as well as single logarithms involving the ratio of the energy to the electroweak scale become dominating. These logarithms arise from virtual (or real) gauge bosons emitted by the initial and final-state particles. They correspond to the well-known soft and collinear singularities observed in QCD.

In the electroweak theory, unlike in massless gauge theories, the large logarithms originating from virtual corrections are of physical significance. In fact, real Z-boson and W-boson bremsstrahlung need not be included, since the masses of the weak gauge bosons, Z and W, provide a physical cutoff, and the massive gauge bosons can be detected as distinguished particles.

The typical size of double-logarithmic (DL) and single-logarithmic (SL) corrections is given by

$$\frac{\alpha}{4\pi s_\mathrm{w}^2}\log^2\frac{s}{M_\mathrm{W}^2} = 6.6\%, \qquad \frac{\alpha}{4\pi s_\mathrm{w}^2}\log\frac{s}{M_\mathrm{W}^2} = 1.3\% \tag{1.1}$$

at $\sqrt{s} = 1\,\mathrm{TeV}$ and increases with energy. If the experimental precision is at the few-percent level like at the LHC, both DL and SL electroweak contributions have to be included at the one-loop level. In view of the precision objectives of a LC, between the percent and the permil level, besides the complete one-loop corrections also two-loop DL effects have to be taken into account.

Owing to this phenomenological relevance, the infrared (IR) structure of the electroweak theory is receiving increasing interest recently. The one-loop structure and the origin of the DL corrections have been discussed for $\mathrm{e^+e^-} \to f\bar{f}$ [6,7] and are by now well established. Recipes for the resummation of the DL corrections have been developed [8,7,9,10] and explicit calculations of the leading DL corrections for the processes $g \to f\bar{f}$ and $\mathrm{e^+e^-} \to f\bar{f}$ have been performed [11,12,13]. On the other hand, for the SL corrections complete one-loop calculations are only available for 4-fermion neutral-current processes [14,9] and W-pair production [4]. The subleading two-loop logarithmic corrections have been evaluated for $\mathrm{e^+e^-} \to f\bar{f}$ in Ref. [9]. A general recipe for a subclass of SL corrections to all orders has been proposed in Ref. [15], based on the infrared-evolution-equation method.

Here, we summarize the results for all DL and SL contributions to the electroweak one-loop virtual corrections published in Ref. [16]. The results apply to exclusive processes with arbitrary external states, including transverse and longitudinal gauge bosons as well as Higgs fields.

The paper is organized as follows: in Section 2 we introduce our notations and discuss the origin of the leading electroweak logarithms. The leading logarithms orig-

inating from the soft–collinear region, from the soft or collinear regions, and from parameter renormalization are considered in Sections 3, 4, and 5, respectively. In Section 6 we apply our general results to W-boson-pair production in $\mathrm{e}^+\mathrm{e}^-$ annihilation.

2 Form and origin of enhanced logarithmic corrections

We consider electroweak processes involving n arbitrary incoming[1] particles (or antiparticles) associated to the fields φ_{i_k},

$$\varphi_{i_1}(p_1)\dots\varphi_{i_n}(p_n) \to 0. \tag{2.1}$$

The indices i_k correspond to the reducible representation of $\mathrm{SU}(2)\times\mathrm{U}(1)$ including all fields in the standard model, and we restrict ourselves to Born matrix elements $\mathcal{M}_0^{i_1\dots i_n}(p_1,\dots,p_n)$ that are not suppressed in the limit where all invariants are much larger than the gauge-boson masses,

$$r_{kl} := (p_k+p_l)^2 \sim 2p_kp_l \gg M_\mathrm{W}^2. \tag{2.2}$$

In the high-energy limit (2.2), we split all enhanced DL and SL corrections into a "symmetric electroweak" (ew) part given by logaritms of the ratio between the energy and the electroweak scale (1.1) and a remaining part that we denote as "pure electromagnetic contribution" (em), which involves logarithms of the light-fermion masses and the infinitesimal photon mass λ used to regularize IR singularities. For the symmetric electroweak logarithms we introduce the shorthands

$$L(s) := \frac{\alpha}{4\pi}\log^2\frac{s}{M_\mathrm{W}^2}, \qquad l(s) := \frac{\alpha}{4\pi}\log\frac{s}{M_\mathrm{W}^2}. \tag{2.3}$$

We assume that the masses M_H, m_t, M_Z, and M_W have the same order of magnitude and neglect all logarithms of ratios of these masses.

In logarithmic approximation (LA) the one-loop corrections to (2.1) assume the form

$$\delta\mathcal{M}^{i_1\dots i_n}(p_1,\dots,p_n) = \mathcal{M}_0^{i'_1\dots i'_n}(p_1,\dots,p_n)\delta_{i'_1i_1\dots i'_ni_n}, \tag{2.4}$$

i.e. they factorize into the lowest-order matrix element times an $\mathrm{SU}(2)\times\mathrm{U}(1)$ matrix. For matrix elements that are not mass-suppressed the factorization formula is universal. The matrix $\delta_{i'_1i_1\dots i'_ni_n}$ can be expressed using the couplings $\mathrm{i}eI^{V_a}(\varphi)$ of the external fields φ_{i_k} to the gauge bosons V_a. These correspond to the generators of infinitesimal global $\mathrm{SU}(2)\times\mathrm{U}(1)$ transformations of these fields,[2]

$$\delta_{V_a}\varphi_i = \mathrm{i}eI^{V_a}_{\varphi_i\varphi_{i'}}(\varphi)\,\varphi_{i'}. \tag{2.5}$$

[1] Usual scattering processes are obtained by crossing symmetry

[2] Details about the explicit form of the generators and other group theoretical quantities can be found in the appendix of Ref. [16].

In terms of the electric charge Q and weak isospin T^a they are given by

$$I^A = -Q, \qquad I^Z = \frac{T^3 - s_\mathrm{w}^2 Q}{s_\mathrm{w} c_\mathrm{w}}, \qquad I^\pm = \frac{T^1 \pm \mathrm{i} T^2}{\sqrt{2} s_\mathrm{w}} \tag{2.6}$$

and depend on the weak mixing angle, which is fixed by $c_\mathrm{w}^2 = 1 - s_\mathrm{w}^2 = M_\mathrm{W}^2/M_\mathrm{Z}^2$.

In general, large logarithms contributing to (2.4) are shared between the loop diagrams and the coupling- and field-renormalization constants, depending on the gauge-fixing and the renormalization scheme. We work within the 't Hooft–Feynman gauge and adopt the on-shell scheme [17] for field and parameter renormalization. We use dimensional regularization and choose the regularization scale $\mu^2 = s$ so that the logarithms $\log(\mu^2/s)$ related to the UV singularities are not enhanced, and only the mass-singular logarithms $\log(\mu^2/M^2)$ or $\log(s/M^2)$ are large. In this setup large logarithms are distributed as follows:

- The DL contributions originate from those one-loop diagrams where soft–collinear gauge bosons are exchanged between pairs of external legs. These double logarithms are obtained with the eikonal approximation.

- The SL mass-singular contributions from loop diagrams originate from the emission of virtual collinear gauge bosons from external lines [18]. These SL contributions are extracted from the loop diagrams in the collinear limit by means of Ward identities, and are found to factorize into the Born amplitude times "collinear factors" [19] .

- The remaining SL contributions originating from soft and collinear regions are contained in the field renormalization constants (FRCs).

- The parameter renormalization (PR) constants, i.e. the charge- and weak-mixing-angle renormalization constants, as well as the renormalization of dimensionless mass ratios associated with the Yukawa and the scalar self-couplings, involve the SL contributions of UV origin. These are the logarithms that are controlled by the renormalization group.

The DL and SL mass-singular terms are extracted from loop diagrams by setting all masses to zero in the numerators of the loop-integrals. For processes involving external longitudinal gauge bosons, this approach is not directly applicable, owing to the longitudinal polarization vectors

$$\epsilon_\mathrm{L}^\mu(p) = \frac{p^\mu}{M} + \mathcal{O}\left(\frac{M}{p^0}\right), \tag{2.7}$$

which are inversely proportional to the gauge boson mass. However, since we are only interested in the high-energy limit, we can use the Goldstone-boson equivalence theorem [20] taking into account the correction factors from higher-order contributions [21].

3 Soft–collinear contributions

The DL corrections originate from loop diagrams where virtual gauge bosons $V_a = \mathrm{A}, \mathrm{Z}, \mathrm{W}^\pm$ are exchanged between pairs of external legs (Figure 1), and arise from the integration region where the gauge-boson momenta are soft and collinear to one of the external legs. They are obtained using the eikonal approximation, and result in a double sum over pairs of external legs

$$\begin{aligned}\delta^{\mathrm{DL}}\mathcal{M}^{i_1\ldots i_n} = {} & \frac{\alpha}{8\pi}\sum_{k=1}^{n}\sum_{l\neq k}\sum_{V_a=A,Z,W^\pm} I^{V_a}_{i'_k i_k}(k) I^{\overline{V}_a}_{i'_l i_l}(l)\mathcal{M}_0^{i_1\ldots i'_k\ldots i'_l\ldots i_n} \\ & \times\left[\log^2\left(\frac{|r_{kl}|}{M^2_{V_a}}\right) - \delta_{V_a A}\log^2\left(\frac{m_k^2}{\lambda^2}\right)\right], \end{aligned} \tag{3.1}$$

where $\overline{V}_a$ represents the charge conjugated of V^a. Formula (3.1) applies to chiral fermions, Higgs bosons, and transverse gauge bosons and depends on their gauge couplings $I^{V_a}(k)$. The DL corrections for external longitudinal gauge bosons $\mathrm{Z_L}$ and $\mathrm{W}^\pm_{\mathrm{L}}$ are obtained from the corrections (3.1) for the corresponding external would-be Goldstone bosons χ and $\phi^\pm$, respectively, using the equivalence theorem

$$\begin{aligned}\delta^{\mathrm{DL}}\mathcal{M}^{\ldots \mathrm{W}^\pm_{\mathrm{L}}\ldots} &= \delta^{\mathrm{DL}}\mathcal{M}^{\ldots\phi^\pm\ldots}, \\ \delta^{\mathrm{DL}}\mathcal{M}^{\ldots \mathrm{Z_L}\ldots} &= \mathrm{i}\delta^{\mathrm{DL}}\mathcal{M}^{\ldots\chi\ldots}. \end{aligned} \tag{3.2}$$

Leading soft–collinear contributions

The DL term in (3.1) containing the invariant r_{kl} depends on the angle between the momenta p_k and p_l. Writing

$$\log^2\left(\frac{|r_{kl}|}{M^2}\right) = \log^2\left(\frac{s}{M^2}\right) + 2\log\left(\frac{s}{M^2}\right)\log\left(\frac{|r_{kl}|}{s}\right) + \log^2\left(\frac{|r_{kl}|}{s}\right), \tag{3.3}$$

one can isolate an angular-independent part proportional to $L(s)$, and this part, together with the additional contributions from photon loops in (3.1), gives the leading

$$\sum_{k=1}^{n}\sum_{l<k}\sum_{V_a=\mathrm{A,Z,W}^\pm}$$

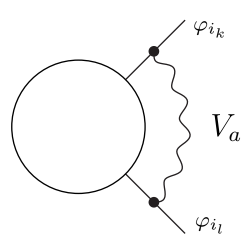


Figure 1: Feynman diagrams leading to DL corrections

soft–collinear (LSC) contribution. Using the invariance of the S matrix with respect to global $\mathrm{SU(2)}\times\mathrm{U(1)}$ transformations, the LSC contribution in (3.1) can be written as a single sum over external legs,

$$\delta^{\mathrm{LSC}}\mathcal{M}^{i_1\dots i_n} = \sum_{k=1}^{n} \delta^{\mathrm{LSC}}_{i'_k i_k}(k)\mathcal{M}_0^{i_1\dots i'_k\dots i_n}, \tag{3.4}$$

where the correction factors reads

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k)L(s) + \delta_{i'_k i_k}Q_k^2 L^{\mathrm{em}}(s,\lambda^2,m_k^2)\right]. \tag{3.5}$$

The first term represents the DL symmetric-electroweak part and is proportional to the effective electroweak Casimir operator[3]

$$C^{\mathrm{ew}} := \sum_{V_a=A,Z,W^\pm} I^{V_a}I^{\overline{V}_a} = \frac{1}{c_{\mathrm{w}}^2}\left(\frac{Y}{2}\right)^2 + \frac{1}{s_{\mathrm{w}}^2}C^{\mathrm{SU(2)}}, \tag{3.6}$$

which depends on the weak hypercharge $Y = 2(Q - T^3)$ and the SU(2) Casimir operator $C^{\mathrm{SU(2)}}$. The second term in (3.5) originates from photon loops and reads

$$L^{\mathrm{em}}(s,\lambda^2,m_k^2) := 2l(s)\log\left(\frac{M_{\mathrm{W}}^2}{\lambda^2}\right) + \frac{\alpha}{4\pi}\left[\log^2\frac{M_{\mathrm{W}}^2}{\lambda^2} - \log^2\frac{m_k^2}{\lambda^2}\right]. \tag{3.7}$$

Subleading soft–collinear contributions

The remaining part of (3.1) is a subleading soft–collinear (SSC) contribution,

$$\delta^{\mathrm{SSC}}\mathcal{M}^{i_1\dots i_n} = \sum_{k=1}^{n}\sum_{l<k}\sum_{V_a=A,Z,W^\pm} \delta^{V_a,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l)\mathcal{M}_0^{i_1\dots i'_k\dots i'_l\dots i_n}. \tag{3.8}$$

This remains a sum over pairs of external legs with angular-dependent factors[4]

$$\delta^{V_a,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l) = \left[2l(s) + \delta_{V_a A}\frac{\alpha}{2\pi}\log\frac{M_{\mathrm{W}}^2}{\lambda^2}\right]\log\frac{|r_{kl}|}{s} I^{V_a}_{i'_k i_k}(k) I^{\overline{V}_a}_{i'_l i_l}(l). \tag{3.9}$$

Owing to the non-diagonal matrices $I^{\pm}(k)$ (cf. appendix of Ref. [16]), the exchange of soft charged gauge bosons involves SU(2)-transformed Born matrix elements on the right-hand side of (3.8).

[3] As explained in Ref. [16] care must be taken for reducible representations, where owing to mixing, (3.6) can be non-diagonal.

[4] Double logarithms of r_{kl}/s are neglected in the limit (2.2).

4 Collinear and soft single logarithms

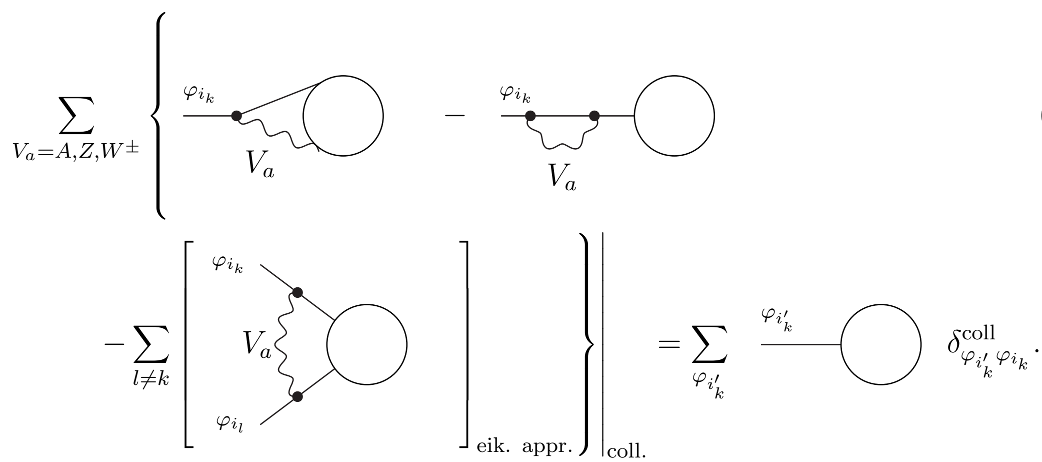
The collinear and soft SL corrections originate from field renormalization and from mass-singular loop diagrams. On one hand the FRCs give the well-known factors $\delta Z_{\varphi}/2$ for each external leg, containing collinear as well as soft SL contributions. On the other hand, mass-singular logarithms arise from the collinear limit of loop diagrams where an external line splits into two internal lines [18], one of these internal lines being a virtual gauge boson A, Z, or W. Both contributions factorize as a sum over the external legs,

$$\delta^{\mathrm{C}}\mathcal{M}^{i_1\ldots i_n}=\sum_{k=1}^{n}\delta^{\mathrm{C}}_{i'_k i_k}(k)\mathcal{M}_0^{i_1\ldots i'_k\ldots i_n} \tag{4.1}$$

with

$$\delta^{\mathrm{C}}_{i'_k i_k}(k)=\frac{1}{2}\delta Z^{\varphi}_{i'_k i_k}+\left.\delta^{\mathrm{coll}}_{i'_k i_k}(k)\right|_{\mu^2=s}. \tag{4.2}$$

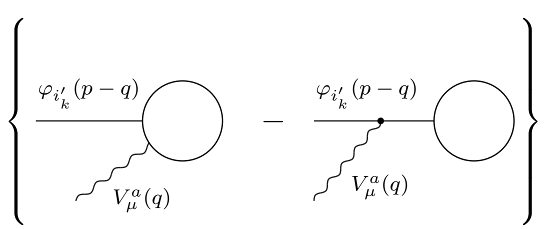
The factorization of the mass-singular loop diagrams will be presented in a forthcoming publication [19]. Therein, we derive the factorization identities



(4.3)

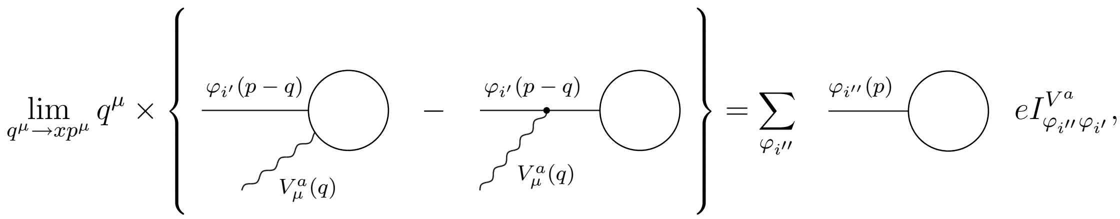
for fermions, gauge bosons and scalar bosons. These identities are obtained by evaluation of the loop diagrams involving the collinear splitting processes $\varphi_{i_k}(p)\to V^a(q)\varphi_{i'_k}(p-q)$, after subtraction of the contributions already contained in the FRCs and the soft collinear corrections. In the limit of collinear gauge-boson emission, the left-hand side of (4.3) is proportional to

$$\sum_{V_a,\varphi_{i'_k}}\int\frac{\mathrm{d}q^D}{(2\pi)^D}\frac{-\mathrm{i}eI^{\overline{V}^a}_{\varphi_{i'_k}\varphi_{i_k}}q^{\mu}}{(q^2-M^2_{V_a})[(p-q)^2-M^2_{\varphi_{i'_k}}]}\times$$



(4.4)

where the diagrams between the curly brackets are contracted with the gauge-boson momentum q^μ. These contractions can be simplified using Ward identities resulting from the BRS symmetry of the spontaneously broken $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge theory (cf. Ref. [19]), and in the limit where q^μ becomes collinear to the external momentum p^μ we obtain



(4.5)

up to mass-suppressed terms. Combining (4.5) with (4.4) we obtain the factorization identity (4.3) with the collinear factors

$$\delta^{\mathrm{coll}}_{\varphi_{i'}\varphi_i} = \frac{\alpha}{4\pi} K \left[C^{\mathrm{ew}}_{\varphi_{i'}\varphi_i} \log \frac{\mu^2}{M_\mathrm{W}^2} + \delta_{\varphi_{i'}\varphi_i} Q^2_{\varphi_i} \log \frac{M_\mathrm{W}^2}{M_{\varphi_i}^2} \right], \tag{4.6}$$

where $K = 2$ for fermions and $K = 1$ for Higgs bosons, would-be Goldstone bosons, and gauge bosons.

In the following, we present the complete SL corrections (4.2) for the cases of external fermions, transverse and longitudinal gauge bosons, and Higgs bosons.

Chiral fermions

For fermions f^κ_σ with chirality $\kappa = \mathrm{R}, \mathrm{L}$ and isospin indices $\sigma = \pm$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'} \left\{ \left[\frac{3}{2} C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8 s_\mathrm{w}^2} \left((1 + \delta_{\kappa\mathrm{R}}) \frac{m^2_{f_\sigma}}{M_\mathrm{W}^2} + \delta_{\kappa\mathrm{L}} \frac{m^2_{f_{-\sigma}}}{M_\mathrm{W}^2} \right) \right] l(s) + Q^2_{f_\sigma} l^{\mathrm{em}}(m^2_{f_\sigma}) \right\}. \tag{4.7}$$

Besides the contribution of the Casimir operator (3.6), we have Yukawa terms proportional to the masses of the fermion f_σ and of its isospin partner $f_{-\sigma}$. These are large for $f^\kappa_\sigma = \mathrm{t}^\mathrm{R}$, t^L, and b^L, where one of the masses is m_t. The pure electromagnetic logarithms are given by

$$l^{\mathrm{em}}(m_f^2) := \frac{\alpha}{4\pi} \left[\frac{1}{2} \log \frac{M_\mathrm{W}^2}{m_f^2} + \log \frac{M_\mathrm{W}^2}{\lambda^2} \right]. \tag{4.8}$$

Transverse physical gauge bosons $A, Z, W^\pm$

The collinear corrections for external physical gauge bosons are related to the one-loop coefficients of the electroweak beta functions. In the mass-eigenstate basis $V_a =$

$A, Z, W^{\pm}$ these coefficients generalize to a matrix b^{ew}_{ab} in the adjoint representation (cf. appendix of Ref. [16]). The charged component reads

$$b^{\mathrm{ew}}_{W^{\sigma}W^{\sigma'}} = \delta_{\sigma\sigma'}\frac{19}{6s_{\mathrm{w}}^2}, \tag{4.9}$$

and determines the running of the SU(2) gauge coupling. In the neutral sector we have

$$b^{\mathrm{ew}}_{AA} = -\frac{11}{3}, \qquad b^{\mathrm{ew}}_{AZ} = b^{\mathrm{ew}}_{ZA} = -\frac{19+22s_{\mathrm{w}}^2}{6s_{\mathrm{w}}c_{\mathrm{w}}}, \qquad b^{\mathrm{ew}}_{ZZ} = \frac{19-38s_{\mathrm{w}}^2-22s_{\mathrm{w}}^4}{6s_{\mathrm{w}}^2c_{\mathrm{w}}^2}. \tag{4.10}$$

The AA component determines the running of the electric charge, and the AZ component is associated with the running of the weak mixing angle [cf. (5.3)]. The SL corrections for transverse gauge bosons are given by

$$\delta^{\mathrm{C}}_{V_aV_b}(V_{\mathrm{T}}) = \frac{1}{2}\left[b^{\mathrm{ew}}_{V_aV_b} + E_{V_aV_b}b^{\mathrm{ew}}_{AZ}\right]l(s) + \delta_{V_aV_b}Q_{V_a}^2 l^{\mathrm{em}}(M_{\mathrm{W}}^2) - \frac{1}{2}\delta_{V_aA}\delta_{V_bA}\Delta\alpha(M_{\mathrm{W}}^2). \tag{4.11}$$

The first term corresponds to the result for a symmetric massless gauge theory like QCD (see for instance Ref. [22]). The second term is proportional to the antisymmetric matrix $E_{V_aV_b}$, with non-vanishing components $E_{AZ} = -E_{ZA} = 1$. This term results from the on-shell renormalization condition [17] and ensures that the correction factor for external photons does not involve mixing with Z bosons,

$$\delta^{\mathrm{C}}_{ZA}(V_{\mathrm{T}}) = 0. \tag{4.12}$$

The third term in (4.11) represents an electromagnetic contribution for charged external gauge bosons. Finally, the AA component receives a pure electromagnetic contribution associated with the light-fermion loops,

$$\Delta\alpha(M_{\mathrm{W}}^2) = \frac{\alpha}{3\pi}\sum_{f,i,\sigma\neq t} N_{\mathrm{C}}^f Q_{f_\sigma}^2 \log\frac{M_{\mathrm{W}}^2}{m_{f_{\sigma,i}}^2} \tag{4.13}$$

where the sum runs over the generations $i = 1, 2, 3$ of leptons and quarks $f = l, q$ with isospin σ and colour factor N_{C}^f, omitting the top-quark contribution.

Longitudinally polarized gauge bosons Z, W$^{\pm}$

The mass singular corrections for external longitudinal gauge bosons Z, W$^{\pm}$, are obtained from the corrections for the corresponding would-be Goldstone bosons χ, $\phi^{\pm}$ using the equivalence theorem. For renormalized amputated Green functions we have the relations

$$\begin{aligned} p^{\mu}\langle W_{\mu}^{\pm}(p)\ldots\rangle &= \pm M_{\mathrm{W}}(1+\delta C_{\phi})\langle\phi_0^{\pm}(p)\ldots\rangle, \\ p^{\mu}\langle Z_{\mu}(p)\ldots\rangle &= \mathrm{i}M_{\mathrm{Z}}(1+\delta C_{\chi})\langle\chi_0(p)\ldots\rangle. \end{aligned} \tag{4.14}$$

Besides the lowest-order contribution, (4.14) contains non-trivial higher-order corrections owing to the mixing between gauge bosons and would-be Goldstone bosons [21]. These corrections correspond to the FRC's for would-be Goldstone bosons in (4.2), and combining them with the collinear factors for would-be Goldstone bosons one obtains

$$\begin{aligned}\delta^{\mathrm{C}}_{\phi^\pm\phi^\pm}(\Phi) &= \delta C_\phi + \delta^{\mathrm{coll}}_{\phi^\pm\phi^\pm}(\Phi) = \left[2C^{\mathrm{ew}}_\Phi - \frac{3}{4s_{\mathrm{w}}^2}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\right] l(s) + Q_{\mathrm{W}}^2 l^{\mathrm{em}}(M_{\mathrm{W}}^2),\\ \delta^{\mathrm{C}}_{\chi\chi}(\Phi) &= \delta C_\chi + \delta^{\mathrm{coll}}_{\chi\chi}(\Phi) = \left[2C^{\mathrm{ew}}_\Phi - \frac{3}{4s_{\mathrm{w}}^2}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\right] l(s).\end{aligned} \tag{4.15}$$

The result is written in terms of the eigenvalue of C^{ew} for the scalar doublet Φ and contains large Yukawa contributions.

Higgs bosons

The SL corrections (4.2) for Higgs bosons read

$$\delta^{\mathrm{C}}_{HH}(\Phi) = \left[2C^{\mathrm{ew}}_\Phi - \frac{3}{4s_{\mathrm{w}}^2}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\right] l(s). \tag{4.16}$$

Note that up to pure electromagnetic contributions, longitudinal gauge bosons and Higgs bosons receive the same collinear SL corrections.

5 Logarithms connected to parameter renormalization

The logarithms related to UV divergences originate from the renormalization of the dimensionless parameters

$$e, \qquad c_{\mathrm{w}} = \frac{M_{\mathrm{W}}}{M_{\mathrm{Z}}}, \qquad h_{\mathrm{t}} = \frac{m_{\mathrm{t}}}{M_{\mathrm{W}}}, \qquad h_{\mathrm{H}} = \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}, \tag{5.1}$$

i.e. the electric charge, the weak mixing angle, and the dimensionless mass ratios related to the top-quark Yukawa coupling and to the scalar self-coupling, respectively. These SL corrections determine the running of the couplings, and in one-loop approximation they are obtained from the Born matrix element $\mathcal{M}_0 = \mathcal{M}_0(e, c_{\mathrm{w}}, h_{\mathrm{t}}, h_{\mathrm{H}})$ in the high-energy limit by

$$\delta^{\mathrm{PR}}\mathcal{M} = \left.\frac{\delta\mathcal{M}_0}{\delta e}\delta e + \frac{\delta\mathcal{M}_0}{\delta c_{\mathrm{w}}}\delta c_{\mathrm{w}} + \frac{\delta\mathcal{M}_0}{\delta h_{\mathrm{t}}}\delta h_{\mathrm{t}} + \frac{\delta\mathcal{M}_0}{\delta h_{\mathrm{H}}}\delta h_{\mathrm{H}}^{\mathrm{eff}}\right|_{\mu^2=s}. \tag{5.2}$$

The contribution from the tadpole renormalization to the renormalization of the scalar self-coupling (cf. Ref. [23]) is included in the effective counterterm $\delta h_{\mathrm{H}}^{\mathrm{eff}}$. In the on-shell scheme (and in LA) the counterterms read

$$
\begin{aligned}
\frac{\delta c_{\mathrm{w}}^2}{c_{\mathrm{w}}^2} &= \frac{s_{\mathrm{w}}}{c_{\mathrm{w}}} b_{AZ}^{\mathrm{ew}} l(\mu^2), \qquad \frac{\delta e^2}{e^2} = -b_{AA}^{\mathrm{ew}} l(\mu^2) + \Delta\alpha(M_{\mathrm{W}}^2),\\
\frac{\delta h_{\mathrm{t}}}{h_{\mathrm{t}}} &= \left\{\frac{1}{2} b_{WW}^{\mathrm{ew}} - \frac{3}{2}\left(C_{\mathrm{t}^{\mathrm{R}}}^{\mathrm{ew}} + C_{\mathrm{t}^{\mathrm{L}}}^{\mathrm{ew}}\right) + \frac{9}{8s_{\mathrm{w}}^2}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\right\} l(\mu^2),\\
\frac{\delta h_{\mathrm{H}}^{\mathrm{eff}}}{h_{\mathrm{H}}} &= \left\{ b_{WW}^{\mathrm{ew}} + \frac{3}{2s_{\mathrm{w}}^2}\left[\frac{M_{\mathrm{W}}^2}{M_{\mathrm{H}}^2}\left(2+\frac{1}{c_{\mathrm{w}}^4}\right) - \left(2+\frac{1}{c_{\mathrm{w}}^2}\right) + \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}\right]\right.\\
&\quad \left. + \frac{3}{s_{\mathrm{w}}^2}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\left(1 - 2\frac{m_{\mathrm{t}}^2}{M_{\mathrm{H}}^2}\right)\right\} l(\mu^2).
\end{aligned}
\tag{5.3}
$$

In the case of processes with longitudinal gauge bosons, the renormalization (5.2) must be performed in the matrix elements resulting from the equivalence theorem.

6 Application to W-boson-pair production

In this section, the above results for Sudakov DL, collinear or soft SL, and PR corrections are applied to W-pair production. Similar results for neutral gauge-boson-pair production and neutral current processes $\mathrm{e}^+\mathrm{e}^- \to f\bar{f}$ can be found in Ref. [16].

We consider the polarized scattering process[5] $\mathrm{e}_\kappa^+\mathrm{e}_\kappa^- \to \mathrm{W}_{\lambda_+}^+\mathrm{W}_{\lambda_-}^-$, where $\kappa = \mathrm{R},\mathrm{L}$ is the electron chirality, and $\lambda_\pm = 0,\pm$ represent the gauge-boson helicities. In the high-energy limit only the following helicity combinations are non-suppressed [4,24]: the purely longitudinal final state $(\lambda_+,\lambda_-) = (0,0)$, which we denote by $\mathrm{W}_{\mathrm{L}}^+\mathrm{W}_{\mathrm{L}}^-$, and the purely transverse and opposite final states $(\lambda_+,\lambda_-) = (\pm,\mp)$, which we denote by $\mathrm{W}_{\mathrm{T}}^+\mathrm{W}_{\mathrm{T}}^-$. The Mandelstam variables are $s = (p_{\mathrm{e}^+} + p_{\mathrm{e}^-})^2$, $t = (p_{\mathrm{e}^+} - p_{\mathrm{W}^+})^2 \sim -s(1-\cos\theta)/2$, and $u = (p_{\mathrm{e}^+} - p_{\mathrm{W}^-})^2 \sim -s(1+\cos\theta)/2$, where θ is the angle between e^+ and W^+. The Born amplitude gets contributions of the s- and t-channel diagrams in Figure 2 and reads

$$
\mathcal{M}_0^{\mathrm{e}_\kappa^+\mathrm{e}_\kappa^- \to \mathrm{W}_{\mathrm{L}}^+\mathrm{W}_{\mathrm{L}}^-} = e^2 R_{\mathrm{e}_\kappa^-\phi^-}\frac{A_s}{s}, \qquad \mathcal{M}_0^{\mathrm{e}_\kappa^+\mathrm{e}_\kappa^- \to \mathrm{W}_{\mathrm{T}}^+\mathrm{W}_{\mathrm{T}}^-} = \delta_{\kappa\mathrm{L}}\frac{e^2}{2s_{\mathrm{w}}^2}\frac{A_t}{t}
\tag{6.1}
$$

up to terms of order M_{W}^2/s, where $R_{\varphi_i\varphi_k} := I_{\varphi_i}^A I_{\varphi_k}^A + I_{\varphi_i}^Z I_{\varphi_k}^Z$ is given by

$$
R_{\mathrm{e}_{\mathrm{R}}^-\phi^-} = \frac{1}{2c_{\mathrm{w}}^2}, \quad R_{\mathrm{e}_{\mathrm{L}}^-\phi^-} = \frac{1}{4s_{\mathrm{w}}^2c_{\mathrm{w}}^2}, \quad R_{\mathrm{e}_{\mathrm{L}}^-\mathrm{W}_{\mathrm{T}}^-} = \frac{1}{2s_{\mathrm{w}}^2}.
\tag{6.2}
$$

The amplitude for transverse gauge-boson production is is non-suppressed only for left-handed electrons in the initial state. In the following we give the one-loop correc-

[5]The momenta and fields of the initial states are incoming, and those of the final states are outgoing.

tions as relative corrections to the Born matrix elements (6.1). The LSC contributions (3.4) read

$$\delta^{\mathrm{LSC}}_{\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to \mathrm{W}^+_\lambda \mathrm{W}^-_{-\lambda}} = -\sum_{\varphi=\mathrm{e}_\kappa,\mathrm{W}_\lambda} \left[C^{\mathrm{ew}}_\varphi L(s) + L^{\mathrm{em}}(s,\lambda^2,m^2_\varphi) \right]. \tag{6.3}$$

Here and in the following formulas, the quantum numbers of the would-be Goldstone bosons $\phi^\pm$ have to be used for longitudinally polarized gauge bosons $\mathrm{W}^\pm_\mathrm{L}$. The eigenvalues of the effective electroweak Casimir operator are

$$C^{\mathrm{ew}}_{\mathrm{e_R}} = \frac{1}{c^2_\mathrm{w}}, \quad C^{\mathrm{ew}}_{\mathrm{e_L}} = C^{\mathrm{ew}}_\Phi = \frac{1+2c^2_\mathrm{w}}{4s^2_\mathrm{w}c^2_\mathrm{w}}, \quad C^{\mathrm{ew}}_{\mathrm{W_T}} = \frac{2}{s^2_\mathrm{w}}. \tag{6.4}$$

The SSC corrections are obtained by applying (3.8) to the crossing symmetric process $\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \mathrm{W}^-_\lambda \mathrm{W}^+_{-\lambda} \to 0$. The contribution of the neutral gauge bosons $V_a = A, Z$ gives

$$\sum_{V_a=A,Z} \delta^{V_a,\mathrm{SSC}}_{\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to \mathrm{W}^+_\lambda \mathrm{W}^-_{-\lambda}} = -\left[4R_{\mathrm{e}_\kappa \mathrm{W}^-_\lambda} l(s) + \frac{\alpha}{\pi} \log \frac{M^2_\mathrm{W}}{\lambda^2} \right] \log \frac{t}{u}, \tag{6.5}$$

The contribution of soft $\mathrm{W}^\pm$ bosons to (3.8) yields

$$\begin{aligned}
\sum_{V_a=W^\pm} \delta^{V_a,\mathrm{SSC}} \mathcal{M}^{\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \phi^- \phi^+} &= \frac{2l(s)\delta_{\kappa\mathrm{L}}}{\sqrt{2}s_\mathrm{w}} \left[\frac{1}{2s_\mathrm{w}} \left(\mathcal{M}_0^{\overline{\nu}_\kappa \mathrm{e}^-_\kappa H \phi^+} + \mathcal{M}_0^{\mathrm{e}^+_\kappa \nu_\kappa \phi^- H} \right) \right. \\
&\quad \left. + \frac{\mathrm{i}}{2s_\mathrm{w}} \left(\mathcal{M}_0^{\overline{\nu}_\kappa \mathrm{e}^-_\kappa \chi \phi^+} - \mathcal{M}_0^{\mathrm{e}^+_\kappa \nu_\kappa \phi^- \chi} \right) \right] \log \frac{|t|}{s}, \\
\sum_{V_a=W^\pm} \delta^{V_a,\mathrm{SSC}} \mathcal{M}^{\mathrm{e}^+_\mathrm{L} \mathrm{e}^-_\mathrm{L} \mathrm{W}^-_\mathrm{T} \mathrm{W}^+_\mathrm{T}} &= \frac{2l(s)}{\sqrt{2}s_\mathrm{w}} \left[\left(\mathcal{M}_0^{\overline{\nu}_\mathrm{L} \mathrm{e}^-_\mathrm{L} A_\mathrm{T} \mathrm{W}^+_\mathrm{T}} + \mathcal{M}_0^{\mathrm{e}^+_\mathrm{L} \nu_\mathrm{L} \mathrm{W}^-_\mathrm{T} A_\mathrm{T}} \right) \right. \\
&\quad \left. - \frac{c_\mathrm{w}}{s_\mathrm{w}} \left(\mathcal{M}_0^{\overline{\nu}_\mathrm{L} \mathrm{e}^-_\mathrm{L} Z_\mathrm{T} \mathrm{W}^+_\mathrm{T}} + \mathcal{M}_0^{\mathrm{e}^+_\mathrm{L} \nu_\mathrm{L} \mathrm{W}^-_\mathrm{T} Z_\mathrm{T}} \right) \right] \log \frac{|t|}{s},
\end{aligned} \tag{6.6}$$

and after explicit evaluation of the SU(2)-transformed Born matrix elements on the left-hand side of (6.6), we find the relative corrections

$$\sum_{V_a=W^\pm} \delta^{V_a,\mathrm{SSC}}_{\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to \mathrm{W}^+_\mathrm{L} \mathrm{W}^-_\mathrm{L}} = -l(s) \frac{\delta_{\kappa\mathrm{L}}}{s^4_\mathrm{w} R_{\mathrm{e}^-_\mathrm{L} \phi^-}} \log \frac{|t|}{s},$$

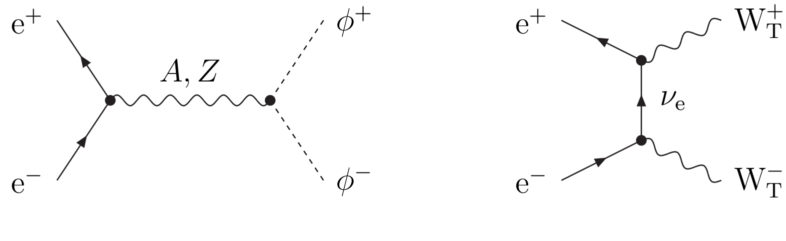


Figure 2: Dominant lowest-order diagrams for $\mathrm{e}^+\mathrm{e}^- \to \phi^+\phi^-$ and $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+_\mathrm{T}\mathrm{W}^-_\mathrm{T}$

$$\sum_{V_a=W^\pm} \delta^{V_a,\mathrm{SSC}}_{\mathrm{e}^+_\mathrm{L}\mathrm{e}^-_\mathrm{L}\to \mathrm{W}^+_\mathrm{T}\mathrm{W}^-_\mathrm{T}} = -\frac{2}{s_\mathrm{w}^2}\left(1-\frac{t}{u}\right) l(s)\log\frac{|t|}{s}. \tag{6.7}$$

The collinear and soft SL corrections can be read off from (4.7), (4.11), and (4.15),

$$\begin{aligned}
\delta^{\mathrm{C}}_{\mathrm{e}^+_\kappa\mathrm{e}^-_\kappa\to \mathrm{W}^+_\mathrm{L}\mathrm{W}^-_\mathrm{L}} &= \left[3C^{\mathrm{ew}}_{\mathrm{e}_\kappa}+4C^{\mathrm{ew}}_{\Phi}\right] l_\mathrm{C} - \frac{3}{2s_\mathrm{w}^2}\frac{m_t^2}{M_\mathrm{W}^2} l_{\mathrm{Yuk}} + \sum_{\varphi=\mathrm{e,W}} 2l^{\mathrm{em}}(m_\varphi^2),\\
\delta^{\mathrm{C}}_{\mathrm{e}^+_\mathrm{L}\mathrm{e}^-_\mathrm{L}\to \mathrm{W}^+_\mathrm{T}\mathrm{W}^-_\mathrm{T}} &= \left[3C^{\mathrm{ew}}_{\mathrm{e}_\mathrm{L}}+b^{\mathrm{ew}}_{WW}\right] l_\mathrm{C} + \sum_{\varphi=\mathrm{e,W}} 2l^{\mathrm{em}}(m_\varphi^2).
\end{aligned} \tag{6.8}$$

Here the Yukawa and non-Yukawa $l(s)$ terms have been denoted by l_{Yuk} and l_C, respectively. Note that the (large) Yukawa contributions occur only for longitudinal gauge bosons.

The PR logarithms are obtained from the renormalization of (6.1). The corresponding $l(s)$ terms are denoted by l_{PR}, and according to (5.3) given by

$$\begin{aligned}
\delta^{\mathrm{PR}}_{\mathrm{e}^+_\mathrm{R}\mathrm{e}^-_\mathrm{R}\to \mathrm{W}^+_\mathrm{L}\mathrm{W}^-_\mathrm{L}} &= -\left[\frac{s_\mathrm{w}}{c_\mathrm{w}} b^{\mathrm{ew}}_{AZ} + b^{\mathrm{ew}}_{AA}\right] l_{\mathrm{PR}} + \Delta\alpha(M_\mathrm{W}^2),\\
\delta^{\mathrm{PR}}_{\mathrm{e}^+_\mathrm{L}\mathrm{e}^-_\mathrm{L}\to \mathrm{W}^+_\mathrm{L}\mathrm{W}^-_\mathrm{L}} &= -\left[(1-\frac{c_\mathrm{w}^2}{s_\mathrm{w}^2})\frac{s_\mathrm{w}}{c_\mathrm{w}} b^{\mathrm{ew}}_{AZ} + b^{\mathrm{ew}}_{AA}\right] l_{\mathrm{PR}} + \Delta\alpha(M_\mathrm{W}^2),\\
\delta^{\mathrm{PR}}_{\mathrm{e}^+_\mathrm{L}\mathrm{e}^-_\mathrm{L}\to \mathrm{W}^+_\mathrm{T}\mathrm{W}^-_\mathrm{T}} &= -b^{\mathrm{ew}}_{WW} l_{\mathrm{PR}} + \Delta\alpha(M_\mathrm{W}^2).
\end{aligned} \tag{6.9}$$

In order to give an impression of the size of the correction, we give a numerical evaluation of the symmetric electroweak part (ew) (2.3) of the above results. Using the physical parameters

$$M_\mathrm{W} = 80.35\,\mathrm{GeV}, \qquad M_\mathrm{Z} = 91.1867\,\mathrm{GeV}, \qquad m_\mathrm{t} = 175\,\mathrm{GeV}, \qquad \alpha = \frac{1}{137.036}, \tag{6.10}$$

we obtain

$$\begin{aligned}
\delta^{\mathrm{ew}}_{\mathrm{e}^+_\mathrm{L}\mathrm{e}^-_\mathrm{L}\to \mathrm{W}^+_\mathrm{T}\mathrm{W}^-_\mathrm{T}} &= -12.6\,L(s) - 8.95\left[\log\frac{t}{u} + \left(1-\frac{t}{u}\right)\log\frac{|t|}{s}\right] l(s) + 25.2\,l_\mathrm{C} - 14.2\,l_{\mathrm{PR}},\\
\delta^{\mathrm{ew}}_{\mathrm{e}^+_\mathrm{L}\mathrm{e}^-_\mathrm{L}\to \mathrm{W}^+_\mathrm{L}\mathrm{W}^-_\mathrm{L}} &= -7.35\,L(s) - \left(5.76\log\frac{t}{u} + 13.9\log\frac{|t|}{s}\right) l(s) + 25.7\,l_\mathrm{C} - 31.8\,l_{\mathrm{Yuk}}\\
&\quad - 9.03\,l_{\mathrm{PR}},\\
\delta^{\mathrm{ew}}_{\mathrm{e}^+_\mathrm{R}\mathrm{e}^-_\mathrm{R}\to \mathrm{W}^+_\mathrm{L}\mathrm{W}^-_\mathrm{L}} &= -4.96\,L(s) - 2.58\left(\log\frac{t}{u}\right) l(s) + 18.6\,l_\mathrm{C} - 31.8\,l_{\mathrm{Yuk}} + 8.80\,l_{\mathrm{PR}}.
\end{aligned} \tag{6.11}$$

These correction factors are shown in Figures 3 and 4 as a function of the scattering angle and the energy, respectively. If the electrons are left-handed, large negative DL and PR corrections originate from the SU(2) interaction. Instead, for right-handed

electrons the DL corrections are smaller, and the PR contribution is positive. For transverse W bosons, there are no Yukawa contributions and the other contributions are in general larger than for longitudinal W bosons. Nevertheless, for energies around 1 TeV, the corrections are similar. Finally, note that the angular-dependent contributions are very important for the LL and LT corrections: at $\sqrt{s} \approx 1\,\mathrm{TeV}$ they vary from $+15\%$ to -5% for scattering angles $30^\circ < \theta < 150^\circ$, whereas the angular-dependent part of the RL corrections remains between $\pm 2\%$.

7 Conclusion

We have considered general electroweak processes at high energies. We have given recipes and explicit formulas for the extraction of the one-loop leading electroweak logarithms. Like the well-known soft–collinear double logarithms, also the collinear single logarithms can be expressed as simple correction factors that are associated with the external particles of the considered process. Up to electromagnetic terms, the collinear SL corrections for external longitudinal gauge bosons and for Higgs bosons are equal. The subleading single logarithms arising from the soft–collinear limit are angular-dependent and can be associated to pairs of external particles. Their

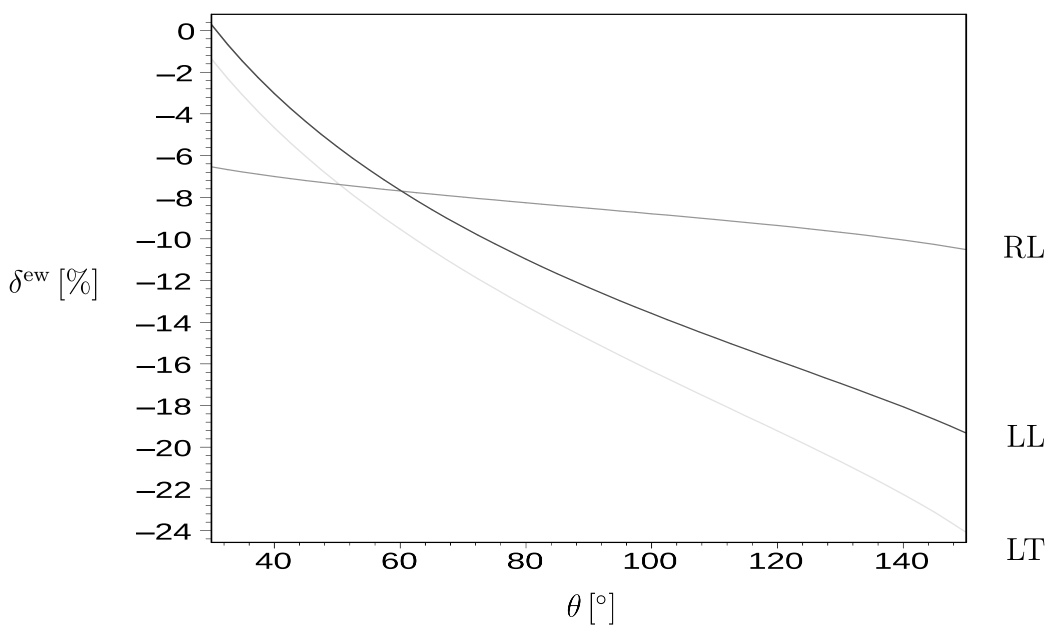


Figure 3: Dependence of the electroweak correction factor $\delta^{\mathrm{ew}}_{\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to \mathrm{W}^+_\lambda \mathrm{W}^-_{-\lambda}}$ on the scattering angle θ at $\sqrt{s} = 1\,\mathrm{TeV}$ for polarizations RL, LL, and LT

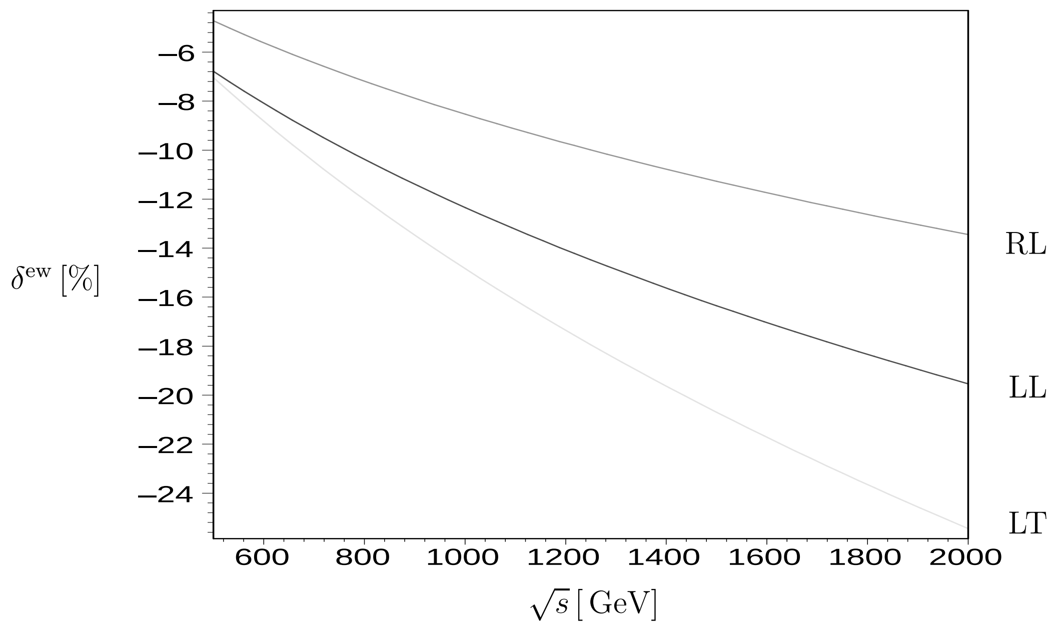


Figure 4: Dependence of the electroweak correction factor $\delta^{\mathrm{ew}}_{\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to \mathrm{W}^+_\lambda \mathrm{W}^-_{-\lambda}}$ on the energy $\sqrt{s}$ at $\theta = 90^\circ$ for polarizations RL, LL, and LT

evaluation requires in general all matrix elements that are linked to the lowest-order matrix element via global SU(2) rotations. Finally, the logarithms originating from coupling-constant renormalization are associated with the explicit dependence of the lowest-order matrix element on the coupling parameters. Our results are applicable to general amplitudes that are not mass-suppressed, as long as all invariants are large compared to the masses. As illustration, we have applied our general results to W-boson-pair production.

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